**Inverse Matrix and Rank of Matrix**

**Inverse Matrix:**

**1. For square matrix:**

Let A and B are two n×n square matrices such that AB=BA=In=1, Then B is said to the inverse of A and we write B=A^-1 and A=B^-1.

**Example 1:**

A= and B=

AB=1

Also BA= 1

**Example 2:**

A=and B=

AB=1

BA=1

**2. For non-square matrix:**

There are two inverse matrix for non-square matrix:

a. Right inverse matrix: If A=(Aij)m×n (m≠n), Then m>n, RTM.

Example

b. Left inverse matrix: If A=(Aij)m×n (m≠n), Then m<n, LTM

Example:

**Adjoint of a square matrix:**

Let A=(aij)n×n is a square matrix, If cofector of aij be Aij, Then (Aij)^T(n×n) is called adjoint of A and is denoted by adj A.

Example 1: A=

Adj A=^T

=^T

=

Example 2: B=

Adj B=

**Singular and Non-Singular Matrices:**

Let A be any square matrix.

If |A|=0,then A is called Singular Matrix.

If |A|≠0 ,then A is called Non-Singular matrix.

**Example-01:**

A=

|A| or Det A==0

So A is a Singular Matrix

**Example-02:**

B=

|B| or Det B=≠0

So B is a Non-Singular Matrix.

**Example-03:**

C=

|C| or Det C=

So C is a Singular Matrix.

**Example-04:**

D=

|D| or Det D=≠0

So D is a Non-Singular Matrix.

**Echelon Matrix:**

Let A=(aij)(m x n) be any matrix. A is said to be an echelon matrix or is said to be in echelon form if

1. Any row consisting entirely of zeros occur at the bottom of the matrix.

2.For two successive non-zero rows, the leading entry in the higher row is further left than the leading entry in the lower row.

3.All the entries in a column below a leading entry are zeroes.

**Example-1:**

A=

This is a 3 by 3 matrix which is in echelon form. Here 3 ,8 and 5 are the leading elements.

**Example-2:**

B=

This is a 4 by 3 matrix which is in echelon form. Here 3,1,7 are the leading elements.

**Example-3:**

C=

This matrix is not in echelon form.

**Equivalent Matrix:**

**Equivalent matrices are matrices whose dimension (or order) are same and corresponding elements within the matrices are equal.**

**Example:**

**A= B=**

**Here both of the matrices A and B has same order and corresponding elements are same.**

**So A and B are equivalent matrices. This is denoted by AB.**

**Rank of a matrix:**

**1.Row Rank:**

**The number of non-zero rows of row echelon-matrix is called row rank.**

**Example-1:**

**A=**

**Here, the row rank is 2 as the first two rows are non-zero row.**

**Example-2:**

**B=**

**Here, the row rank is 1 as only the first row is a non-zero row.**

**2.Column Rank:**

**The number of non-zero column of column echelon-matrix is called column rank.**

**Example-1:**

**A=**

**Here, the column rank is 2 as the first two columns are non-zero column.**

**Example-2:**

**B=**

**Here, the column rank is 1 as only the first column is a non-zero column.**

**Row Rank or Column Rank of a matrix is called the rank of the matrix. Row rank and column rank of every matrix is same.**

**Example: Find the row rank and column rank of the following matrix**

**Solution:**

= ,=

= =

It is now row echelon matrix.it has 3 non-zero rows so row rank of the matrix is 3.

**Column echelon matrix:**

= ,=,=,=

= =,=,=

= ==

It is now column echelon matrix.it has 3 non-zero column so column rank of the matrix is 3.

**Find the row rank and column rank of the following matrices:**

1. 2.

3.

**Example: Find the rank of the following matrices(either row or column operation):**

1. 2. = 2

3.  4. = 2

**Theory-1:** Let A=(Aij)nxn is any matrix. Prove that A-1 = adj / |A|, where A is a non-singular matrix.

**Prof:** We Know, A (adj A) = |A| In

* A-1A (adj A)=|A|A-1In
* In (adj A) = |A|A-1In
* A-1 = adj A /|A|
* Proved.

**Example:** If A= Then find out adj A and A-1

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**Solution:** adj A = ------------- ( I )

Here, A11= = -1; A12= = 0; A13= = -1; A21= = 0;

A22= = 0; A23= = -1; A31= = 1; A32= = -2; A33= = 1;

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From (1), adj A = =

Now, A == 1(-1) + 1(-1) =-2

= > A-1 = adj A / |A| = - 1/ 2

**Find the inverse Matrix of the following** **Matrix:**

**Example 01:**

A =

**Solution:**

|A|=

=4-(-2)+1

=4(0-6)+2(30+3)+1(10-0)

=-24+66+10

=52

Now, we will determine the ad joint of the matrix A by calculating the cofactors of each element and then taking the transpose of the cofactor matrix.

Adj A=

The inverse of matrix A is given by the formula A-1 =( 1/|A|).Adj A

A-1=(1/52).

=

Answer:

**Find the inverse matrix of the following:**

**Example 02:**

**A=**

A-1 =(1/-23).

**Example 03:**

A=

A-1 =(1/28).

**Example 04:**

A=

A-1 =(1/3).

**Eigen Values and Eigen Vector**

**Definition**: Let A be any n order square matrix over the field F. If there any exist λ ∈ F and a non-zero column vector V ∈ Fn such that Av = λv,

Then the scaler λ is called eigen value and the non-zero column vector corresponding λ be called eigen-vector.

Let us have a look at the example given below to learn how to find the eigenvalues of a 2 x 2 matrix.

**Note that:**

Also let I be a unit or identity matrix of square order. Then,

**(i)** λI-A is called **Characteristic Matrix.**

**(ii)** The determinant |λI-A| of the characteristic matrix λI-A is called **Characteristic Polynomial.**

**(iii)** |λI-A| = 0 is called **Characteristic Equation.**

**Example:** Find the eigenvalues of the 2 x 2 matrix

A=

**Solution:**

Given,

A=

Let

I=

**(i) Characteristic Matrix:** λI-A

=> λ

=>

**(ii) Characteristic Polynomial:**  | λI-A |

=> -λ2 - 3λ-4

=>(λ+1) (λ-4)

**(ii) Characteristic Equation:**  | λI-A |=0

* (λ+1) (λ – 4) = 0
* λ+1 = 0 or λ – 4 = 0

Thus, λ = -1 or λ = 4

Hence, the two eigenvalues of the given matrix are λ = -1 and λ = 4.

Now, **Eigen Vector,**

Let column vector V corresponding to λ is:

V=

* ( λI-A )V=0
* --------(i)

-2x+(-----(ii)

For

From equation (i) and (ii) we get

-3x-3y=0-------(iii)

-2x-2y=0-------(iv)

Solving (iii) and (iv)

x=-1, y=1

Thus for , eigen vector is

ii) for λ=4,

(1) becomes 2x-3y=0

~ 2x-3y = 0

=> y=2, x=3

-2x+3y=0 \\

∴ for λ=4, eigenvector:

Therefore, Eigen value of Matrix A is: -1,4

Eigen vector of Matrix A is:

**Example:** Find the eigen values and eigen vectors of the matrix-

A =

**Solution:**

1. Characteristic Matrix:

λI - A = λ =

1. Characteristic polynomial:

= =

1. Characteristic equation:

= 0

=> = 0

=> λ=2, -5

؞ Eigen values of matrix A is: 2, -5

Now, Eigen vector

Let column vector v corresponding to λ is:

v=

Now, = 0

=>

=>

….…………..(1)

1. For =2,
2. becomes

-3x + 6y =0

Or, y =

Or, y =

∴ for λ=2, eigenvector:

1. For = -5 ,

(1)becomes

3x + y =0

Or, x =

Or, x =

∴ for λ= -5, eigenvector:

Therefore, Eigen value of Matrix A is: 2,-5

Eigen vectors of Matrix A is:

**Exercise:**

Find all eigen values and associated eigen vector of the following Matrix.

1. A = (ii) (iii)

(iv) A = (v) A = (vi) A =